

An Effective Lagrangian with Broken Scale and Chiral Symmetry IV: Nucleons and Mesons at Finite Temperature

G.W. Carter[†] and P.J. Ellis

School of Physics and Astronomy
University of Minnesota, Minneapolis, MN 55455

Abstract

We study the finite temperature properties of an effective chiral Lagrangian which describes nuclear matter. Thermal fluctuations in both the nucleon and the meson fields are considered. The logarithmic and square root terms in the effective potential are evaluated by expansion and resummation with the result written in terms of the exponential integral and the error function, respectively. In the absence of explicit chiral symmetry breaking a phase transition restores the symmetry, but when the pion has a mass the transition is smooth. The nucleon and meson masses as a functions of density and temperature are discussed.

PACS: 11.10.Wx, 12.39.Fe, 24.10.Jv

Keywords: effective Lagrangian, finite temperature, chiral symmetry restoration

[†] Present address: Niels Bohr Institute, University of Copenhagen, Denmark.

1 Introduction

In previous work [1, 2], hereinafter referred to as I and II respectively, we have described an effective Lagrangian which incorporates broken scale symmetry in addition to spontaneously broken $SU(2)$ chiral symmetry, as suggested by quantum chromodynamics (QCD). This Lagrangian contained a potential with logarithmic terms involving the glueball field ϕ and the chiral σ and π fields. At temperature $T = 0$ this led to a good description of nuclear matter and finite nuclei at the mean field level as well as low energy πN scattering data. The extension of this type of model to $SU(3)$ has recently been discussed by Papazoglou *et al.* [3]. Here we examine the predictions of our $SU(2)$ Lagrangian for the finite temperature, $T > 0$, properties of hadronic matter which are needed in astrophysical applications and in the study of relativistic heavy ion collisions. Previous studies [4, 5] of models of this general type at $T > 0$ have simply included temperature effects for the nucleons. Clearly thermal effects for the mesons are also significant, particularly those due to the pion which will be dominant at low temperatures.

The analysis at finite temperature is not straightforward due to the logarithmic terms in the potential. We have recently suggested in Ref. [6], hereinafter referred to as III, a method of treating these terms which involves expansion and resummation with the result cast in terms of the exponential integral. This technique was applied to the meson sector of the effective Lagrangian. Here we want to carry out a more complete calculation by including nucleons as well. As we shall show, this involves a square root term which, after analogous treatment, gives results which can be written in terms of the error function. The Lagrangian and our thermal analysis are discussed in Section 2, with the detailed expressions for the necessary thermal averages being relegated to the Appendix. We give our numerical results in Section 3 and Section 4 contains our conclusions.

2 Theory

2.1 Equations of Motion

We will simplify the Lagrangian given in II by excluding the isotriplet vector and axial vector mesons, the ρ and a_1 . They give no mean field contribution to symmetric nuclear matter and, since they are relatively heavy, their thermal fluctuations will not play a significant role in chiral symmetry restoration. We can exclude an additional term which was introduced to obtain the physical value of the axial coupling constant, g_A , since it involves the quantity $\bar{N}\boldsymbol{\tau}N$ which will not contribute for symmetric nuclear matter. We also discard a term $\epsilon_3\bar{N}N$, which explicitly breaks chiral symmetry, since in II tiny values of ϵ_3 in the range 0 to -15 MeV were preferred. An unfavored symmetry breaking term labelled ϵ_2 is also omitted. Then our effective Lagrangian can be written

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma + \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \frac{1}{2}G_{\omega\phi}\phi^2\omega_\mu\omega^\mu \\ & + [(G_4)^2\omega_\mu\omega^\mu]^2 + \bar{N}\left[\gamma^\mu(i\partial_\mu - g_\omega\omega_\mu) - g\sqrt{\sigma^2 + \boldsymbol{\pi}^2}\right]N - \mathcal{V}, \\ \mathcal{V} = & B\phi^4\left(\ln\frac{\phi}{\phi_0} - \frac{1}{4}\right) - \frac{1}{2}B\delta\phi^4\ln\frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2} + \frac{1}{2}B\delta\zeta^2\phi^2\left[\sigma^2 + \boldsymbol{\pi}^2 - \frac{\phi^2}{2\zeta^2}\right] \\ & - \frac{1}{4}\epsilon'_1\left(\frac{\phi}{\phi_0}\right)^2\left[\frac{4\sigma}{\sigma_0} - 2\left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2}\right) - \left(\frac{\phi}{\phi_0}\right)^2\right] - \frac{3}{4}\epsilon'_1.\end{aligned}\tag{1}$$

Here $\zeta = \frac{\phi_0}{\sigma_0}$ and in the vacuum $\phi = \phi_0$, $\sigma = \sigma_0$ and $\boldsymbol{\pi} = 0$, regardless of whether or not the explicit symmetry breaking term ϵ'_1 is present. The field strength tensor for the ω field is $\omega_{\mu\nu} = \partial_\mu\omega_\nu - \partial_\nu\omega_\mu$. The ω mass is generated by coupling to the glueball field as was previously found to be necessary in order to describe nuclei. The mass term can be written in terms of the vacuum mass $m_\omega = G_{\omega\phi}^{1/2}\phi_0$. In Eq. (1) we have also included a quartic term in the ω field since this generally improved the phenomenology in I and II.

In Eq. (1) we have replaced the traditional coupling of the pion and the sigma

meson to the nucleon, $-\bar{N}g(\sigma + i\boldsymbol{\pi} \cdot \boldsymbol{\tau}\gamma_5)N$, by $-\bar{N}g\sqrt{\sigma^2 + \boldsymbol{\pi}^2}N$, which is a more natural form for including thermal fluctuations. As pointed out by Weinberg [7] such a transformation can be achieved by a redefinition of the nucleon field. The form of the ω coupling is invariant, while the extra terms generated by the derivative involve $\bar{N}\boldsymbol{\tau}N$ and so will not contribute here. (The transformation has no effect if the pion field is set to zero as appropriate to $T = 0$.)

The quantities B and δ in Eq. (1) are parameters. For the latter, guided by the QCD beta function, we take $\delta = 4/33$ as in previous work. The logarithmic terms contribute to the trace anomaly: in addition to the standard contribution from the glueball field [8, 9] there is a contribution from the σ field. There is also a contribution from the explicit symmetry breaking, which is related to the pion mass. Specifically the trace of the “improved” energy-momentum tensor is $\theta_\mu^\mu = 4\epsilon_{\text{vac}}(\phi/\phi_0)^4$, where the vacuum energy $\epsilon_{\text{vac}} = -\frac{1}{4}B\phi_0^4(1 - \delta) - \epsilon'_1$.

We take the vacuum glueball mass to be approximately 1.5 GeV in view of QCD sum rule estimates [10] of 1.5 GeV and recent lattice estimates [11] of 1.7 GeV; small shifts in the precise value of the mass are inconsequential here. Since the mass is large in comparison to the temperatures of interest, we shall neglect thermal effects for the glueball. We define the ratio of the mean field to the vacuum value to be $\chi = \phi/\phi_0$. Then the thermal averages of Lagrange’s equations for the ϕ , σ and ω fields in uniform matter are:

$$\begin{aligned}
0 &= 4B_0\chi^3 \ln \chi - B_0\delta\chi \left\langle 2\chi^2 \ln \left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2} \right) - \left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2} \right) \right\rangle - B_0\delta\chi^3 \\
&\quad - \epsilon'_1\chi \left\langle \frac{2\sigma}{\sigma_0} - \left(\frac{\sigma^2 + \boldsymbol{\pi}^2}{\sigma_0^2} \right) \right\rangle + \epsilon'_1\chi^3 - m_\omega^2\chi \langle \omega_\mu \omega^\mu \rangle , \\
0 &= -B_0\sigma_0^2\delta\chi^4 \left\langle \frac{\sigma}{\sigma^2 + \boldsymbol{\pi}^2} \right\rangle + (B_0\delta\chi^2 + \epsilon'_1)\langle \sigma \rangle - \epsilon'_1\chi^2\sigma_0 + \left\langle \frac{g\sigma_0^2\sigma}{\sqrt{\sigma^2 + \boldsymbol{\pi}^2}} \bar{N}N \right\rangle , \\
0 &= -m_\omega^2\chi^2 \langle \omega_\mu \rangle - 4G_4^4 \langle \omega_\nu \omega^\nu \omega_\mu \rangle + g_\omega \langle \bar{N}\gamma_\mu N \rangle ,
\end{aligned} \tag{2}$$

where we have defined $B_0 = B\phi_0^4$. Here the thermal averages are denoted by angle brackets.

Consider first thermal effects for the σ and π fields. We break σ into a mean field part $\bar{\sigma}$ and a fluctuation $\Delta\sigma$ with mean value $\langle\Delta\sigma\rangle = 0$. The mean value of the pion field $\langle\pi\rangle$ is, of course, zero. We write

$$\begin{aligned}\frac{\sigma^2 + \pi^2}{\sigma_0^2} &= \frac{1}{\sigma_0^2}(\bar{\sigma}^2 + 2\bar{\sigma}\Delta\sigma + \Delta\sigma^2 + \pi^2) \\ &\equiv \nu^2 + 2\nu\Delta\nu + \psi^2,\end{aligned}\tag{3}$$

where, as in III, $\nu = \bar{\sigma}/\sigma_0$, $\Delta\nu = \Delta\sigma/\sigma_0$ and $\psi^2 = (\Delta\sigma^2 + \pi^2)/\sigma_0^2$. Since expanding the fluctuations out to lowest order will not properly treat $(\sigma^2 + \pi^2)$ when it occurs in the denominator or in the logarithm in Eqs. (2), we proceed as in III by expanding out the cross term $2\nu\Delta\nu$ of Eq. (3). The motivation is that at low temperatures the thermal average of ψ^2 is small, while at high temperatures ν is small, so the cross term is relatively small in both limits. Note that odd powers of $\Delta\nu$ can be dropped since the thermal average gives zero. We also have to contend with products of meson and baryon factors. For these we adopt a factorization ansatz, *i.e.*, $\langle f(\sigma, \pi)\bar{N}N \rangle \simeq \langle f(\sigma, \pi) \rangle \langle \bar{N}N \rangle$. For the omega field we write $\omega_\mu = \omega_0 + \Delta\omega_\mu$ and terms linear in $\Delta\omega_\mu$ will vanish. The G_4 term, which involves four ω fields, will be treated at mean field level only, since it is a small correction and since this avoids the complication of differing longitudinal and transverse masses. With these approximations Eqs. (2) become

$$\begin{aligned}0 &= B_0\delta\chi \left\langle -2\chi^2 \ln(\nu^2 + \psi^2) + \frac{4\chi^2\nu^2\Delta\nu^2}{(\nu^2 + \psi^2)^2} + \frac{8\chi^2\nu^4\Delta\nu^4}{(\nu^2 + \psi^2)^4} + \psi^2 \right\rangle + B_0\delta\chi\nu^2 \\ &\quad + B_0\chi^3(4\ln\chi - \delta) - \epsilon'_1\chi(2\nu - \nu^2 - \langle\psi^2\rangle - \chi^2) - m_\omega^2\chi(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle), \\ 0 &= B_0\delta\chi^4\nu \left\langle -\frac{1}{\nu^2 + \psi^2} + \frac{2\Delta\nu^2}{(\nu^2 + \psi^2)^2} - \frac{4\nu^2\Delta\nu^2}{(\nu^2 + \psi^2)^3} + \frac{8\nu^2\Delta\nu^4}{(\nu^2 + \psi^2)^4} \right\rangle \\ &\quad + g\sigma_0\nu\rho_S \left\langle \frac{1}{\sqrt{\nu^2 + \psi^2}} - \frac{\Delta\nu^2}{(\nu^2 + \psi^2)^{3/2}} + \frac{3\nu^2\Delta\nu^2}{2(\nu^2 + \psi^2)^{5/2}} - \frac{5\nu^2\Delta\nu^4}{2(\nu^2 + \psi^2)^{7/2}} \right\rangle\end{aligned}$$

$$\begin{aligned}
& +B_0\delta\chi^2\nu - \epsilon'_1\chi^2(1-\nu) , \\
0 = & -m_\omega^2\chi^2\omega_0 - 4G_4^4\omega_0^3 + g_\omega\rho .
\end{aligned} \tag{4}$$

Notice that there is only a solution with exact chiral symmetry, $\nu = 0$, when there is no explicit symmetry breaking, $\epsilon'_1 = 0$. For most purposes it is sufficient to truncate these equations one power lower, however in the absence of explicit symmetry breaking this has the disadvantage that there is a small region near the chiral phase transition where solutions cannot be obtained. The expansion parameter is of order $4\nu^2\langle\Delta\nu^2\rangle/(\nu^2 + \langle\psi^2\rangle)^2$ which *a posteriori* we find to be < 0.05 – this is satisfactorily small.

The evaluation of the thermal average of the logarithm and the integral powers of $(\nu^2 + \psi^2)$ was discussed in detail in III. A formal expansion in ψ^2 was made. The thermal average of $(\psi^2)^n$ was then written in terms of $\langle\psi^2\rangle^n$ using counting factors that assume $\langle\psi_i^2\rangle$ is independent of i , that is $\langle\Delta\sigma^2\rangle = \langle\pi_a^2\rangle$ where π_a is a component of the pion field. This is a high temperature approximation which will be accurate when chiral symmetry is exactly or approximately restored or when thermal contributions are dominant. Nevertheless, our expressions also yield the correct low temperature limit. This suggests that our approximation is reasonable, although we do not have a quantitative assessment of the errors involved in the intermediate region. The final step is to resum the series and the result can be written in terms of the exponential integral. A similar procedure is followed for the half-integral powers for which the result can be expressed in terms of the error function. This is discussed in Appendix A and expressions are listed there for all the thermal averages needed in Eqs. (4) and the equations below. Note that the $T = 0$ limit of Eqs. (4) yields the relativistic mean field expressions given in II.

The evaluation of $\langle\psi^2\rangle$ requires the thermal average of the square of a scalar or

pseudoscalar field. The standard results are

$$\langle \pi_a^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty dk \frac{k^2}{e_\pi} \frac{1}{e^{\beta e_\pi} - 1} \quad ; \quad \langle \Delta \sigma^2 \rangle = \frac{1}{2\pi^2} \int_0^\infty dk \frac{k^2}{e_\sigma} \frac{1}{e^{\beta e_\sigma} - 1} . \quad (5)$$

Here $\beta = 1/T$ is the inverse temperature and π_a represents a component of the pion field. The energies, $e_\pi = \sqrt{k^2 + m_\pi^{*2}}$ and $e_\sigma = \sqrt{k^2 + m_\sigma^{*2}}$, depend upon m_π^{*2} and m_σ^{*2} , respectively the effective pion and sigma masses. Here the asterisks denote the finite temperature and/or density masses which will be calculated in the next subsection. For the vector ω the corresponding result is

$$\langle \Delta \omega_\mu \Delta \omega^\mu \rangle = -\frac{3}{2\pi^2} \int_0^\infty dk \frac{k^2}{e_\omega} \frac{1}{e^{\beta e_\omega} - 1} , \quad (6)$$

with $e_\omega = \sqrt{k^2 + m_\omega^{*2}}$. The nucleon density and the scalar density in Eqs. (4) are

$$\begin{aligned} \rho &= \frac{2}{\pi^2} \int_0^\infty dk k^2 \left(\frac{1}{e^{\beta(E^* - \mu^*)} + 1} - \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right) , \\ \rho_S &= \frac{2M^*}{\pi^2} \int_0^\infty dk \frac{k^2}{E^*} \left(\frac{1}{e^{\beta(E^* - \mu^*)} + 1} + \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right) , \end{aligned} \quad (7)$$

where the effective chemical potential $\mu^* = \mu - g_\omega \omega_0$ and the energy $E^* = \sqrt{k^2 + M^{*2}}$ with the nucleon effective mass M^* defined below. In calculating thermodynamic integrals, such as these, we find it convenient to make use of the numerical approximation scheme of Ref. [12].

2.2 Masses

Consistent with our factorization hypothesis, we define the effective nucleon mass to be

$$\begin{aligned} M^* &= g \left\langle \sqrt{\sigma^2 + \boldsymbol{\pi}^2} \right\rangle \\ &= g\sigma_0 \left\langle \sqrt{\nu^2 + \boldsymbol{\psi}^2} - \frac{\nu^2 \Delta \nu^2}{2(\nu^2 + \boldsymbol{\psi}^2)^{3/2}} - \frac{5\nu^4 \Delta \nu^4}{8(\nu^2 + \boldsymbol{\psi}^2)^{7/2}} \right\rangle , \end{aligned} \quad (8)$$

where we have truncated the expansion at the same order as the equation of motion. When $T = 0$, Eq. (A.5) indicates that the usual result $M^* = g\sigma_0\nu$ is obtained, with the vacuum nucleon mass, $M = g\sigma_0$, determining g for a given value of σ_0 . Since we consider thermal fluctuations, the nucleon mass will not become zero when chiral symmetry is restored and $\nu \rightarrow 0$. In fact,

$$\lim_{\nu \rightarrow 0} \frac{M^*}{M} = \sqrt{\frac{9\pi\langle\psi^2\rangle}{32}}. \quad (9)$$

In the case where $\epsilon'_1 = 0$ the sigma and pion are massless at the chiral restoration temperature T_c , so $\langle\psi^2\rangle = T_c^2/(3\sigma_0^2)$ and $M^*/M = 0.543T_c/\sigma_0$.

For the meson fields we define the effective mass at finite temperature in terms of the thermal average of the second derivative of the Lagrangian. This means that we only consider contributions arising from a single interaction vertex. Since the mixing between the glueball and the σ meson is small, we neglect it here for simplicity. Specifically

$$\begin{aligned} \sigma_0^2 m_\sigma^{*2} &= -\sigma_0^2 \left\langle \frac{\partial^2 \mathcal{L}}{\partial \Delta \sigma^2} \right\rangle = (B_0\delta + \epsilon'_1)\chi^2 + \left\langle -\frac{B_0\delta\chi^4\sigma_0^2}{\sigma^2 + \pi^2} + \frac{2B_0\delta\chi^4\sigma_0^2\sigma^2}{(\sigma^2 + \pi^2)^2} \right\rangle \\ &\quad + g\sigma_0^2\rho_S \left\langle \frac{1}{\sqrt{\sigma^2 + \pi^2}} - \frac{\sigma^2}{(\sigma^2 + \pi^2)^{3/2}} \right\rangle, \\ \sigma_0^2 m_\pi^{*2} &= -\sigma_0^2 \left\langle \frac{\partial^2 \mathcal{L}}{\partial \pi_a^2} \right\rangle = (B_0\delta + \epsilon'_1)\chi^2 + \left\langle -\frac{B_0\delta\chi^4\sigma_0^2}{\sigma^2 + \pi^2} + \frac{2B_0\delta\chi^4\sigma_0^2\pi_a^2}{(\sigma^2 + \pi^2)^2} \right\rangle \\ &\quad + g\sigma_0^2\rho_S \left\langle \frac{1}{\sqrt{\sigma^2 + \pi^2}} - \frac{\pi_a^2}{(\sigma^2 + \pi^2)^{3/2}} \right\rangle, \\ m_\omega^{*2} &= \left\langle \frac{\partial^2 \mathcal{L}}{\partial \Delta \omega_\mu \partial \Delta \omega^\mu} \right\rangle = m_\omega^2 \chi^2, \\ \phi_0^2 m_\phi^{*2} &= -\phi_0^2 \left\langle \frac{\partial^2 \mathcal{L}}{\partial \phi^2} \right\rangle = 4B_0\chi^2(3\ln\chi + 1) + 3(\epsilon'_1 - B_0\delta)\chi^2 + (B_0\delta + \epsilon'_1)\nu^2 \\ &\quad - 2\epsilon'_1\nu + \left\langle -6B_0\delta\chi^2 \ln\left(\frac{\sigma^2 + \pi^2}{\sigma_0^2}\right) + (B_0\delta + \epsilon'_1)\psi^2 \right\rangle \\ &\quad - m_\omega^2(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle). \end{aligned} \quad (10)$$

Notice that at zero temperature a nucleon contribution to the pion mass of $g\sigma_0\rho_S/\nu$ is automatically obtained. In II this required the evaluation of the nucleon loop contribution to the pion propagator, whereas here it arises from the form of the coupling used for the chiral meson fields and the nucleon. As we have remarked, we do not consider thermal fluctuations in the glueball field and so its mass does not enter the equations. However it will be useful to display the mass in Sec. 3. The σ and π masses are needed in evaluating $\langle\psi^2\rangle$ and the ω mass depends on χ . As a result, the equations of motion (4) and the expressions for the masses (10) must be evaluated self-consistently.

We need to expand the denominators in Eqs. (10) as discussed in the previous subsection. The expressions can be simplified by using the equations of motion (4) for the case where $\nu \neq 0$ and $\chi \neq 0$, as well as the relation

$$\frac{1}{\sigma_0} \left\langle \frac{\sigma}{(\sigma^2 + \boldsymbol{\pi}^2)^\alpha} \right\rangle = \nu \left\langle \frac{1}{(\sigma^2 + \boldsymbol{\pi}^2)^\alpha} \right\rangle - 2\alpha\nu \left\langle \frac{\pi_a^2}{(\sigma^2 + \boldsymbol{\pi}^2)^{\alpha+1}} \right\rangle, \quad (11)$$

which is valid in our approximation scheme for α an arbitrary integer or half-integer. We obtain

$$\begin{aligned} \sigma_0^2 m_\sigma^{*2} &= 2B_0\delta\chi^4\nu^2 \left\langle \frac{1}{(\nu^2 + \boldsymbol{\psi}^2)^2} - \frac{8\Delta\nu^2}{(\nu^2 + \boldsymbol{\psi}^2)^3} + \frac{4(3\nu^2\Delta\nu^2 + 2\Delta\nu^4)}{(\nu^2 + \boldsymbol{\psi}^2)^4} \right\rangle + \frac{\epsilon'_1\chi^2}{\nu} \\ &\quad - g\sigma_0\rho_S\nu^2 \left\langle \frac{1}{(\nu^2 + \boldsymbol{\psi}^2)^{3/2}} - \frac{6\Delta\nu^2}{(\nu^2 + \boldsymbol{\psi}^2)^{5/2}} + \frac{5(3\nu^2\Delta\nu^2 + 2\Delta\nu^4)}{2(\nu^2 + \boldsymbol{\psi}^2)^{7/2}} \right\rangle, \\ \sigma_0^2 m_\pi^{*2} &= \frac{\epsilon'_1\chi^2}{\nu}, \\ \phi_0^2 m_\phi^{*2} &= 4(B_0\chi^2 + \epsilon'_1\nu) - 2(B_0\delta + \epsilon'_1)(\nu^2 + \langle\boldsymbol{\psi}^2\rangle) + 2m_\omega^2(\omega_0^2 + \langle\Delta\omega_\mu\Delta\omega^\mu\rangle). \end{aligned} \quad (12)$$

It is straightforward to verify that in the zero temperature limit the results of II are obtained. The explicit symmetry breaking parameter $\epsilon'_1 = (\sigma_0 m_\pi)^2$ is fixed by the vacuum pion mass and the chosen value of σ_0 . For the case when $\nu \rightarrow 0$ the σ and π

masses become equal:

$$\sigma_0^2 m_\sigma^{*2} \rightarrow \sigma_0^2 m_\pi^{*2} \rightarrow B_0 \delta \chi^2 \left(1 - \frac{\chi^2}{\langle \psi^2 \rangle} \right) + g \sigma_0 \rho_S \sqrt{\frac{9\pi}{32 \langle \psi^2 \rangle}} + \epsilon'_1 \chi^2 . \quad (13)$$

When $\epsilon'_1 = 0$, the expression on the right is valid for temperatures at which ν is exactly zero.

2.3 Thermodynamics

The grand potential per unit volume can easily be written down:

$$\begin{aligned} \frac{\Omega}{V} = & \langle \mathcal{V} \rangle - \frac{1}{2} m_\omega^{*2} \chi^2 \omega_0^2 - G_4^4 \omega_0^4 - \frac{1}{2} m_\sigma^{*2} \langle \Delta \sigma^2 \rangle - \frac{1}{2} m_\pi^{*2} \langle \pi^2 \rangle \\ & + \frac{T}{2\pi^2} \int dk k^2 \left[\ln(1 - e^{-\beta e_\sigma}) + 3 \ln(1 - e^{-\beta e_\pi}) + 3 \ln(1 - e^{-\beta e_\omega}) \right] \\ & - \frac{2T}{\pi^2} \int dk k^2 \left[\ln \left(1 + e^{-\beta(E^* - \mu^*)} \right) + \ln \left(1 + e^{-\beta(E^* + \mu^*)} \right) \right] . \end{aligned} \quad (14)$$

The subtraction of the fourth and fifth terms on the right in Eq. (14) is necessary to avoid double counting [13].

Now if one takes the partial derivative of Ω/V with respect to χ , ν , or ω_0 the equations of motion (2) are obtained. This is an important and non-trivial consistency check. In order to show this one needs the equivalences

$$\begin{aligned} \frac{\partial}{\partial \langle \psi^2 \rangle} \left\langle \ln \left(\frac{\sigma^2 + \pi^2}{\sigma_0^2} \right) \right\rangle &= \frac{1}{2} \left\langle \frac{\sigma_0^2}{\sigma^2 + \pi^2} \right\rangle , \\ \frac{\partial}{\partial \langle \psi^2 \rangle} \left\langle \frac{\sqrt{\sigma^2 + \pi^2}}{\sigma_0} \right\rangle &= \frac{3}{8} \left\langle \frac{\sigma_0}{\sqrt{\sigma^2 + \pi^2}} \right\rangle . \end{aligned} \quad (15)$$

We have not succeeded in proving these relations in general, but by expanding out $2\nu\Delta\nu$ as before and using the explicit equations in the Appendix we have verified Eqs. (15) to orders $(\nu^2 + \psi^2)^{-4}$ and $(\nu^2 + \psi^2)^{-9/2}$, respectively, in our approximation scheme. This is all that is needed here. Furthermore Eqs. (15) are exact in the limits of zero and infinite temperature.

In practice it is necessary to truncate the expansions. If the equations for the fields, the masses and the grand potential are truncated at the same order, then the pion mass is exactly zero in the absence of explicit symmetry breaking and there is approximate consistency between the equations of motion and the grand potential. Alternatively the equations can be truncated at different orders such that there is exact consistency between the equations of motion and the grand potential, but then the pion mass is non-zero at low temperature when $\epsilon'_1 = 0$, thus violating Goldstone's theorem. We choose the former alternative and note that the inaccuracy is not large, although it can result in a small negative value for the pressure at low temperature.

We therefore truncate the expansion for the thermal average of the potential at order $(\nu^2 + \psi^2)^{-4}$ and write

$$\begin{aligned} \langle \mathcal{V} \rangle = & \chi^4 [B_0 \ln \chi - \tfrac{1}{4} B_0 (1 + \delta) + \tfrac{1}{4} \epsilon'_1] + \tfrac{1}{2} (B_0 \delta + \epsilon'_1) \chi^2 (\nu^2 + \langle \psi^2 \rangle) \\ & - \epsilon'_1 \chi^2 \nu - \tfrac{1}{2} B_0 \delta \chi^4 \left\langle \ln(\nu^2 + \psi^2) - \frac{2\nu^2 \Delta \nu^2}{(\nu^2 + \psi^2)^2} - \frac{4\nu^4 \Delta \nu^4}{(\nu^2 + \psi^2)^4} \right\rangle \\ & + \tfrac{1}{4} [B_0 (1 - \delta) + \epsilon'_1] . \end{aligned} \quad (16)$$

We have added a constant term here so that $\langle \mathcal{V} \rangle$ is zero in the vacuum. The pressure P is of course $-\Omega/V$.

It is straightforward to derive the energy density, which takes the form

$$\begin{aligned} \frac{E}{V} = & \langle \mathcal{V} \rangle + \tfrac{1}{2} m_\omega^{*2} \chi^2 \omega_0^2 + 3G_4^4 \omega_0^4 - \tfrac{1}{2} m_\sigma^{*2} \langle \Delta \sigma^2 \rangle - \tfrac{1}{2} m_\pi^{*2} \langle \pi^2 \rangle \\ & + \frac{1}{2\pi^2} \int dk k^2 \left[\frac{e_\sigma}{e^{\beta e_\sigma} - 1} + \frac{3e_\pi}{e^{\beta e_\pi} - 1} + \frac{3e_\omega}{e^{\beta e_\omega} - 1} \right] \\ & + \frac{2}{\pi^2} \int dk k^2 E^* \left(\frac{1}{e^{\beta(E^* - \mu^*)} + 1} + \frac{1}{e^{\beta(E^* + \mu^*)} + 1} \right) . \end{aligned} \quad (17)$$

Table 1: Values of the parameters.

Quantity	$G_4 = 0$	$G_4/g_\omega = 0.19$
$ \epsilon_{\text{vac}} ^{1/4}$ (MeV)	236	228
g_ω	10.5	12.2
$\zeta = \phi_0/\sigma_0$	1.28	1.41
σ_0 (MeV)	110	102
$\epsilon_1'^{1/4}$ (MeV)	123	119

3 Results

The parameters used to obtain the numerical results are listed in Table 1. They were determined in II by fitting to the properties of nuclear matter and finite nuclei. Quite a reasonable phenomenology was obtained there, with the non-zero value of G_4 being slightly favored, so it is sensible to explore the behavior at $T > 0$. We shall also consider the case where the vacuum pion mass vanishes ($\epsilon_1' = 0$) for which the parameters differ only slightly from those specified in Table 1.

If $\epsilon_1' = 0$, so that the pion is massless in the vacuum, chiral symmetry can be exactly restored at sufficiently high temperature. We show in Fig. 1 the chiral restoration temperature T_c versus the ratio of baryon density ρ to the density at saturation ρ_0 . The primary distinction between the zero and non-zero G_4 cases arises from the different values of σ_0 in Table 1, which fix the overall temperature scale. Thus the same qualitative behavior is seen in the two cases with the corresponding temperatures slightly larger when $G_4 = 0$; at zero density this scaling factor is almost exactly $110/102 \approx 1.1$, but it increases to about 1.7 at very high density. Therefore it is sufficient in the following to focus on the $G_4 \neq 0$ results.

In III, baryons and ω mesons were absent and T_c was 187 MeV. Since at restoration $m_\sigma^* = m_\pi^* = 0$, Eq. (13) shows that T_c is modified by the scalar density which is non-zero even at zero baryon density. We obtain a value of 162 MeV, which is a reasonable order of magnitude in view of the many estimates in the literature. We mention that the standard Gell-Mann-Lévy model [14] yields $T_c = \sqrt{2}f_\pi = 132$ MeV. Also Gerber and Leutwyler [15] estimate $T_c = 190$ MeV in two-flavor QCD using an effective chiral Lagrangian to three loop order, while a recent lattice calculation [16] gave 140 – 150 MeV. The restoration temperature decreases with increasing density so that at ρ_0 it is 125 MeV and asymptotically it is 45 MeV. A finite T_c is required in the high density limit since at $T = 0$, as shown in Fig. 2, ν becomes $\simeq 0.3$, which is small but non-zero. Thus chiral symmetry is not completely restored at zero temperature in our model; this finds some support from lattice calculations [17]. Papazoglou *et al.* [5] have found that there are regions in parameter space for models of this type where chiral symmetry is restored at high density, but with these potentials the compression modulus is unreasonably high.

The remainder of Fig. 2 shows that ν becomes zero at progressively lower densities as the temperature is raised. The dotted lines here indicate metastable regions and, while the transition is clearly first order, we do not believe that our approximation scheme is sufficiently accurate to reliably determine the order. For instance, if our expansions are truncated at a lower order we are unable to find solutions for a small region in the vicinity of the transition. Suppressing results in the metastable regions, we plot the sigma mass in Fig. 3. For $T = 0$ the mass increases with density due to the scalar density contribution in Eq. (12). With increasing temperature the familiar behavior is seen, namely, the mass drops to zero to become degenerate with the pion mass at successively lower densities. Beyond this point the two masses increase together as either the temperature or density is raised.

We now turn to the more realistic case where $\epsilon'_1 \neq 0$ and the pion has its physical mass in the vacuum. Figure 4 plots ν as a function of density. At low temperatures ν increases slightly for finite densities, before decreasing as the temperature is further raised and becoming rather independent of density. Substantial reduction in ν requires higher temperatures than needed for Fig. 2, which seems reasonable in view of the finite pion mass here. With baryons present ν smoothly tends towards zero as $T \rightarrow \infty$, whereas in their absence it stabilized at $\simeq 0.15$, as demonstrated in III. In either case there is no phase transition when $\epsilon'_1 \neq 0$. In this connection it is worth mentioning that the lattice QCD calculations of Brown *et al.* [18] show that a second order phase transition for massless quarks is washed out when they are given a finite mass.

The pion and sigma masses are shown in Figs. 5 and 6. As the temperature is raised the pion mass increases, while the sigma mass decreases so that they approach each other. They are degenerate when a temperature of ~ 250 MeV is reached and depend very little on density. Thereafter they continue to rise, passing the 1 GeV mark at $T \simeq 300$ MeV (not illustrated). The sigma mass remains above 400 MeV for all densities and temperatures. This is qualitatively different from the high temperature behavior for the $\epsilon'_1 = 0$ case shown in Fig. 3. Furthermore, comparison with III indicates that the presence of baryons pushes up the mass noticeably.

The nucleon effective mass, M^* , is shown in Fig. 7. At low temperatures its behavior is dominated by ν and the mass increases slightly with temperature for non-zero densities. However, the thermal fluctuations, $\langle \psi^2 \rangle$, begin to dominate for temperatures above 150 MeV, leading to a drop in M^* at low densities. This thermal domination implies that the behavior of the mean field ν is of minor consequence, which is why a plot of M^* for the $\epsilon'_1 = 0$ case looks quite similar to Fig. 7. By $T = 250$ MeV, M^* is rather independent of density. The mass continues to fall at higher temperatures and remains roughly independent of density. At such high T this is not caused by the

very small ν , but rather by the diminished value of $\langle\psi^2\rangle$ which follows from the sharp increase in the meson masses seen in Figs. 5 and 6. Qualitatively the overall behavior of M^* is strikingly similar to that reported by Furnstahl and Serot [19] for the Walecka model. For a given T , their mass drops to a lower level than ours, probably because they did not consider meson fluctuations. On the other hand at least one model – the quark meson coupling model – shows a different behavior [20] in that the nucleon mass simply increases monotonically with temperature.

The ratio of the glueball mean field to the vacuum value, $\chi = \phi/\phi_0$, is shown in Fig. 8. This ratio differs from unity by more than 10% only for the highest temperature, 250 MeV, and low to moderate densities. To complete the discussion, we must examine the glueball effective mass of Eq. (12), m_ϕ^* , which is displayed in Fig. 9. Again we remark that this mass was not used in the self-consistent solution of the equations since glueball fluctuations were excluded on the grounds that the mass is large. This is proven justified for low temperatures, but it is becoming questionable at $T = 250$ MeV for low density. Also the difference between m_ϕ^* and m_σ^* starts to become small and it may not be adequate to ignore the mixing induced by the off-diagonal terms in the mass matrix described in II (the vacuum mass in Fig. 9 will be pushed up a little by this mixing). Furthermore, beyond this temperature the fluctuations of the ω meson field, $\langle\Delta\omega_\mu\Delta\omega^\mu\rangle$, start to become significant. This reduces the glueball effective mass further and continuing to solve for m_ω^* and χ self-consistently quickly sends m_ϕ^* to zero at $T \sim 300$ MeV. This effect was not observed in III since the ω meson was not included. To obtain reliable solutions in this region one would first of all need to consider fluctuations in the glueball field and compute the mixing with the σ field. Secondly, since the mass of the ρ meson is similar to that of the ω , the fluctuations in this field will also play a role even though its mean field is zero. In addition one should include the chiral partner of the ρ , namely the a_1 , since with the approximate

restoration of chiral symmetry the masses will be similar. If the effect of fluctuations in the ρ and a_1 fields is estimated by appropriately increasing the degeneracy factor for the ω fluctuations, it is found that m_ϕ^* is driven to zero at a temperature of ~ 200 MeV (at zero density; the effect is softened with increasing density, but still present). Although we expect that glueball fluctuations will partially counter this effect, it is intriguing since it could be an intimation of deconfinement. Lattice calculations suggest that this occurs at a similar temperature to chiral restoration [21]. However a consistent thermal analysis for the ϕ , ρ and a_1 mesons would be quite involved and is beyond the scope of the present work.

Turning to the thermodynamics of our model, we first remark that in common with other models we observe a liquid-gas phase transition; the critical temperature is found to be 16 MeV. The pressure and energy density are plotted in Figs. 10 and 11, respectively, as functions of density. The calculations here are for $\epsilon'_1 \neq 0$, but taking $\epsilon'_1 = 0$ yields similar results, except that all numbers are reduced by a few percent in the absence of a finite pion mass. At the highest temperatures and densities shown, it can be noted that $E \sim 3PV$, which is the massless gas result. However this does not hold for lower values of either T or ρ , for which interactions dominate the thermal fluctuations. For orientation we have compared the pressure to that obtained with a crude lowest order treatment of gluons and massless quarks, taking the bag constant to be $|\epsilon_{\text{vac}}|$. We find that at no point, either in temperature or density, is the quark-gluon plasma the preferred phase prior to chiral restoration (for $\epsilon'_1 = 0$). At zero density, for example, chiral restoration is achieved at $T_c = 162$ MeV while the quark-gluon plasma has a larger pressure for $T \simeq 170$ MeV. At equilibrium density, where we found $T_c = 125$ MeV, the quark-gluon phase is preferred at $T = 164$ MeV.

4 Conclusions

We have discussed the finite temperature behavior of an effective Lagrangian with which we have successfully described nuclear matter and finite nuclei at $T = 0$. With nucleons present, we have thermally averaged a square root term involving the meson fields in addition to a logarithmic term. The latter was handled in III by expansion and resummation of an infinite series with the final result cast in terms of the exponential integral. For the former the result of a similar approach was written in terms of the error function.

Our results showed that at sufficiently high temperature the mean value of the σ field became small, signalling chiral restoration. In the absence of explicit chiral symmetry breaking, a phase transition restored the symmetry at temperatures ranging from 162 MeV at zero density to 45 MeV at very large density. It was estimated that that the quark-gluon phase would be preferred at somewhat higher temperatures than these. In the physical case, where explicit chiral symmetry breaking was present and the pion had a vacuum mass, a smooth restoration of chiral symmetry was found. The masses of the pion and σ meson were virtually degenerate and the order parameter ν became small for temperatures of ~ 250 MeV. At such a temperature the nucleon mass was also reduced, but the effect of meson fluctuations yielded some stabilization so that $M^*/M \simeq 0.43$. At a similar temperature the glueball mass started to be significantly reduced due to the coupling to ω vector meson fluctuations. This could be a hint of interesting physics, but we were unable to track it further with the present formalism since glueball fluctuations and the ρ and a_1 mesons are expected to be of significance.

We thank S. Rudaz for useful discussions. We acknowledge partial support from the Department of Energy under grant No. DE-FG02-87ER40328. G.W.C. thanks the University of Minnesota for a Doctoral Dissertation Fellowship. A grant for computing time from the Minnesota Supercomputer Institute is gratefully acknowledged.

Appendix A. Thermal Averages

A.1. Half-Integral Powers

We first write the expansion

$$\left\langle \sqrt{\nu^2 + \psi^2} \right\rangle = \nu \left\langle 1 + \frac{\psi^2}{2\nu^2} - \sum_{n=2}^{\infty} (-1)^n \frac{(2n-3)!!}{2^n n!} \frac{\psi^{2n}}{\nu^{2n}} \right\rangle. \quad (\text{A.1})$$

For the purposes of evaluating the counting, we assume that the thermal average $\langle \psi_i^2 \rangle$ is independent of the label i , which amounts to assuming the masses of the particles involved are the same. Then the result of taking the thermal average of each possible pair of fields at a general vertex can be written $\langle (\psi^2)^n \rangle = c_n \langle \psi^2 \rangle^n$ and, as shown in III, $c_n = (n+1)!/2^n$. Using this result and defining

$$\frac{1}{z^2} = \frac{\langle \psi^2 \rangle}{2\nu^2}, \quad (\text{A.2})$$

we have

$$\left\langle \sqrt{\nu^2 + \psi^2} \right\rangle = \nu \left[1 + \frac{1}{z^2} + \sum_{m=1}^{\infty} (-1)^m \frac{(m+2)(2m-1)!!}{2^{m+1}} z^{-2(m+1)} \right]. \quad (\text{A.3})$$

This is a divergent series which we regard as a formal expansion and it must be re-summed before it can be evaluated. Matching this expression to the asymptotic expansions of $i^n \text{erfc}(z)$, the repeated integrals of the complementary error function [22], we have

$$\begin{aligned} \left\langle \sqrt{\nu^2 + \psi^2} \right\rangle &= \nu + \frac{\nu}{4z} \sqrt{\pi} e^{z^2} \left[2zi^1 \text{erfc}(z) + 3i^0 \text{erfc}(z) \right] \\ &= \frac{3\nu}{4z} \left[2z + \left(1 - \frac{2}{3}z^2\right) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right]. \end{aligned} \quad (\text{A.4})$$

Here $\text{erfc}(z) = 1 - 2\pi^{-\frac{1}{2}} \int_0^z e^{-t^2} dt$. In the limit of low temperature ($z \rightarrow \infty$)

$$\left\langle \sqrt{\nu^2 + \psi^2} \right\rangle \rightarrow \nu + \frac{\langle \psi^2 \rangle}{2\nu}, \quad (\text{A.5})$$

as follows from expanding out the square root directly. In the high temperature limit ($z \rightarrow 0$)

$$\left\langle \sqrt{\nu^2 + \psi^2} \right\rangle \rightarrow \frac{3}{4} \sqrt{\frac{\pi}{2}} \sqrt{\langle \psi^2 \rangle}, \quad (\text{A.6})$$

and the numerical factor is 0.9400 which is close to unity, as one might expect.

The other expressions that are needed can be obtained by differentiating with respect to ν^2 and by using the relations

$$\begin{aligned} \left\langle \Delta \nu^2 f(\nu, \psi^2) \right\rangle &= \frac{1}{4} \left\langle \psi^2 f(\nu, \psi^2) \right\rangle, \\ \left\langle \Delta \nu^4 f(\nu, \psi^2) \right\rangle &= 3\sigma_0^{-2} \left\langle \Delta \nu^2 \pi_a^2 f(\nu, \psi^2) \right\rangle = \frac{1}{8} \left\langle \psi^4 f(\nu, \psi^2) \right\rangle, \end{aligned} \quad (\text{A.7})$$

which are easily shown for an arbitrary function f , when $\langle \psi_i^2 \rangle$ is independent of i . One obtains the following:

$$\begin{aligned} \left\langle (\nu^2 + \psi^2)^{-\frac{1}{2}} \right\rangle &= \frac{z}{2\nu} \left[2z + (1 - 2z^2) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right], \\ \left\langle (\nu^2 + \psi^2)^{-\frac{3}{2}} \right\rangle &= -\frac{z^3}{\nu^3} \left[2z - (1 + 2z^2) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right], \\ \left\langle \Delta \nu^2 (\nu^2 + \psi^2)^{-\frac{3}{2}} \right\rangle &= \frac{z}{8\nu} \left[2z + 4z^3 + (1 - 4z^2 - 4z^4) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right], \\ \left\langle \Delta \nu^2 (\nu^2 + \psi^2)^{-\frac{5}{2}} \right\rangle &= -\frac{z^3}{12\nu^3} \left[10z + 4z^3 - (3 + 12z^2 + 4z^4) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right], \\ \left\langle \Delta \nu^2 (\nu^2 + \psi^2)^{-\frac{7}{2}} \right\rangle &= \frac{z^4}{30\nu^5} \left[8 + 18z^2 + 4z^4 \right. \\ &\quad \left. - (15z + 20z^3 + 4z^5) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right], \\ \left\langle \Delta \nu^4 (\nu^2 + \psi^2)^{-\frac{7}{2}} \right\rangle &= -\frac{z^3}{120\nu^3} \left[66z + 56z^3 + 8z^5 \right. \\ &\quad \left. - (15 + 90z^2 + 60z^4 + 8z^6) \sqrt{\pi} e^{z^2} \text{erfc}(z) \right]. \end{aligned} \quad (\text{A.8})$$

A.2. Integral Powers

The thermal averages needed can be derived in the manner indicated above using the thermal average of the logarithm. This is evaluated in analogous fashion to the

square root and the details were discussed in III. We simply list the results here, in terms of the variable $y = z^2$:

$$\begin{aligned}
\langle \ln(\nu^2 + \psi^2) \rangle &= \ln \nu^2 + (1 - y)e^y E_1(y) + 1 , \\
\langle (\nu^2 + \psi^2)^{-1} \rangle &= \frac{y}{\nu^2} [1 - ye^y E_1(y)] , \\
\langle (\nu^2 + \psi^2)^{-2} \rangle &= -\frac{y^2}{\nu^4} [1 - (1 + y)e^y E_1(y)] , \\
\langle (\nu^2 + \psi^2)^{-3} \rangle &= \frac{y^2}{2\nu^6} [1 + y - (2y + y^2)e^y E_1(y)] , \\
\langle \Delta \nu^2 (\nu^2 + \psi^2)^{-2} \rangle &= \frac{y}{4\nu^2} [1 + y - (2y + y^2)e^y E_1(y)] , \\
\langle \Delta \nu^2 (\nu^2 + \psi^2)^{-3} \rangle &= -\frac{y^2}{8\nu^4} [3 + y - (2 + 4y + y^2)e^y E_1(y)] , \\
\langle \Delta \nu^2 (\nu^2 + \psi^2)^{-4} \rangle &= \frac{y^2}{24\nu^6} [2 + 5y + y^2 - (6y + 6y^2 + y^3)e^y E_1(y)] , \\
\langle \Delta \nu^4 (\nu^2 + \psi^2)^{-4} \rangle &= -\frac{y^2}{48\nu^4} [11 + 8y + y^2 \\
&\quad - (6 + 18y + 9y^2 + y^3)e^y E_1(y)] . \tag{A.9}
\end{aligned}$$

The exponential integral is defined [22] by $E_1(y) = \int_1^\infty dt t^{-1} e^{-yt}$.

References

- [1] E.K. Heide, S. Rudaz and P.J. Ellis, Nucl. Phys. **A571** (1994) 713.
- [2] G. Carter, P.J. Ellis and S. Rudaz, Nucl. Phys. **A603** (1996) 367; erratum **A608** (1996) 514.
- [3] P. Papazoglou, S. Schramm, J. Schaffner-Bielich, H. Stöcker and W. Greiner, nucl-th/9706024.
- [4] G. Kälbermann, J.M. Eisenberg and B. Svetitsky, Nucl. Phys. **A600** (1996) 436.

- [5] P. Papazoglou, J. Schaffner, S. Schramm, D. Zschesche, H. Stöcker and W. Greiner, Phys. Rev. **C55** (1997) 1499.
- [6] G.W. Carter, P.J. Ellis and S. Rudaz, Nucl. Phys. **A618** (1997) 317.
- [7] S. Weinberg, The Quantum Theory of Fields, Vol. II, (Cambridge University Press, 1996) p. 202.
- [8] J. Schechter, Phys. Rev. **D21** (1980) 3393; A.A. Migdal and M.A. Shifman, Phys. Lett. **B114** (1982) 445.
- [9] H. Gomm and J. Schechter, Phys. Lett. **B158** (1985) 449.
- [10] M.A. Shifman, Z. Phys. **C9** (1981) 347; P. Pascual and R. Tarrach, Phys. Lett. **B113** (1982) 495.
- [11] J. Sexton, A. Vaccarino and D. Weingarten, Phys. Rev. Lett. **75** (1995) 4563.
- [12] S.M. Johns, P.J. Ellis and J.M. Lattimer, Astrophys. J. **473** (1996) 1020.
- [13] T.D. Lee and M. Margulies, Phys. Rev. **D11** (1975) 1591.
- [14] A. Bochkarev and J. Kapusta, Phys. Rev. **D54** (1996) 4066.
- [15] P. Gerber and H. Leutwyler, Nucl. Phys. **B321** (1989) 387.
- [16] C. Bernard, T. Blum, C. DeTar, S. Gottlieb, K. Rummukainen, U.M. Heller, J.E. Hetrick, D. Toussaint, L. Kärkkäinen, B. Sugar and M. Wingate, Phys. Rev. **D55** (1997) 6861.
- [17] T. Blum, J.E. Hetrick and D. Toussaint, Phys. Rev. Lett. **76** (1996) 1019.
- [18] F.R. Brown, F.P. Butler, H. Chen, N.H. Christ, Z. Dong, W. Schaffer, L.I. Unger and A. Vaccarino, Phys. Rev. Lett. **65** (1990) 2491.

- [19] R.J. Furnstahl and B.D. Serot, Phys. Rev. **C41** (1990) 262.
- [20] P.K. Panda, A. Mishra, J.M. Eisenberg and W. Greiner, nucl-th/9705045.
- [21] H. Satz, hep-ph/9611366, Proceedings of the First Catania Relativistic Ion Studies (Acicastello, Italy, 1996)p. 208.
- [22] M. Abramowitz and I.A. Stegun, eds. Handbook of Mathematical Functions (Dover, NY, 1965).

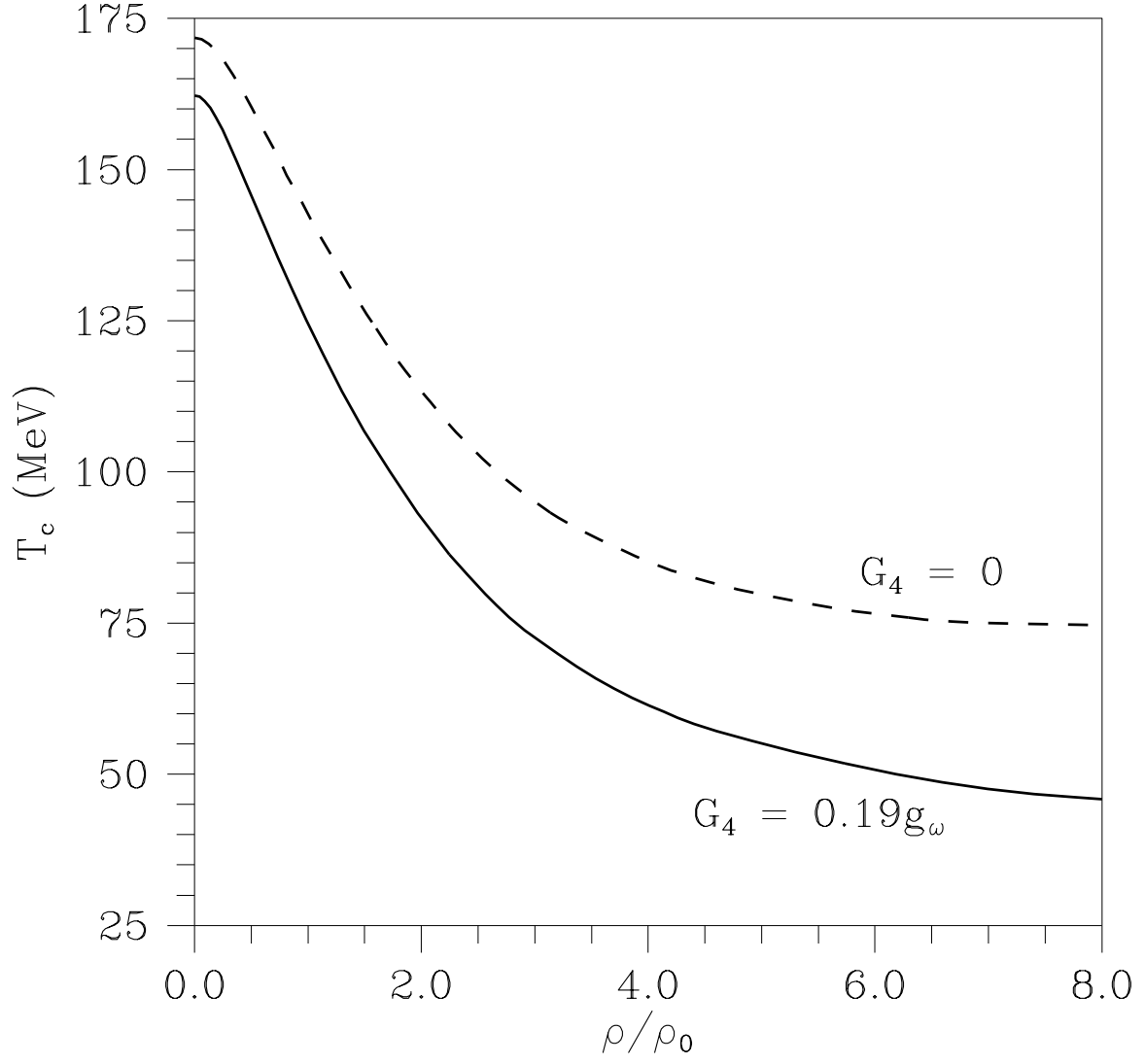


Figure 1. The chiral restoration temperature T_c as a function of the ratio of the density to equilibrium nuclear matter density, ρ/ρ_0 , for two values of G_4 with $\epsilon'_1 = 0$.

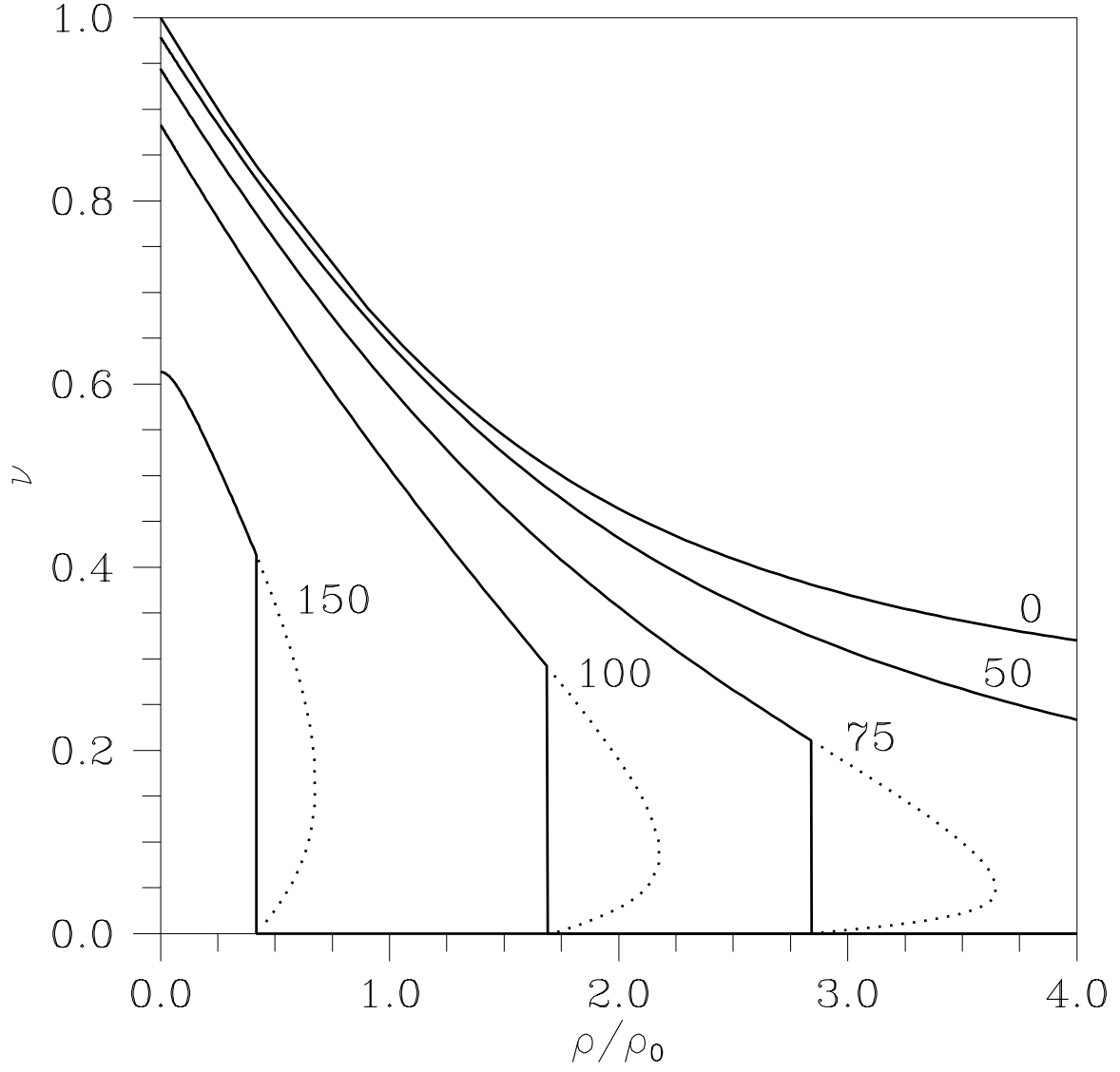


Figure 2. The mean sigma field, $\nu = \bar{\sigma}/\sigma_0$, as a function of density for various temperatures (in MeV) with $\epsilon'_1 = 0$. The dotted curves indicate thermodynamically unstable regions.

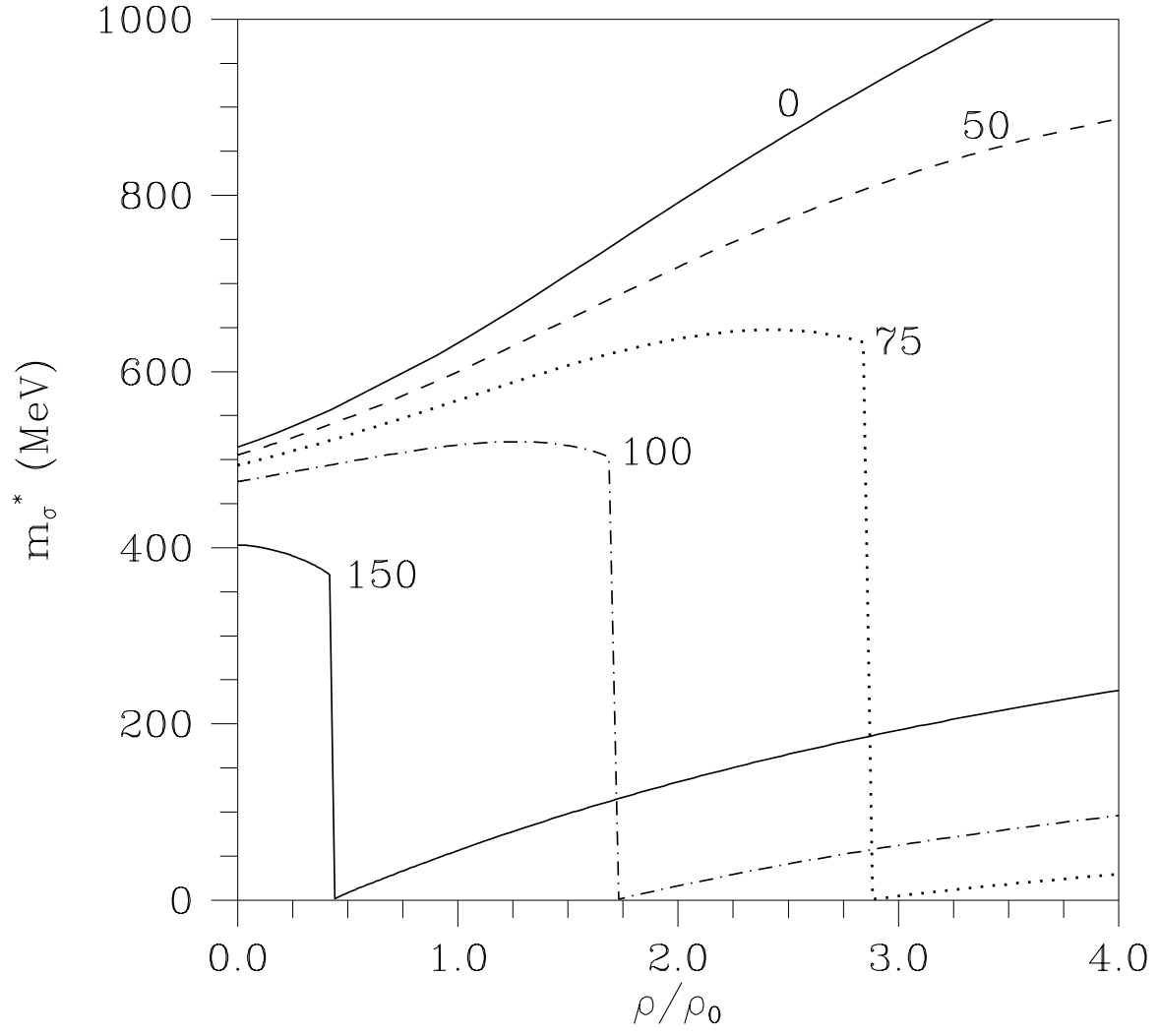


Figure 3. The sigma effective mass, m_σ^* , as a function of density for various temperatures (in MeV) with $\epsilon'_1 = 0$.

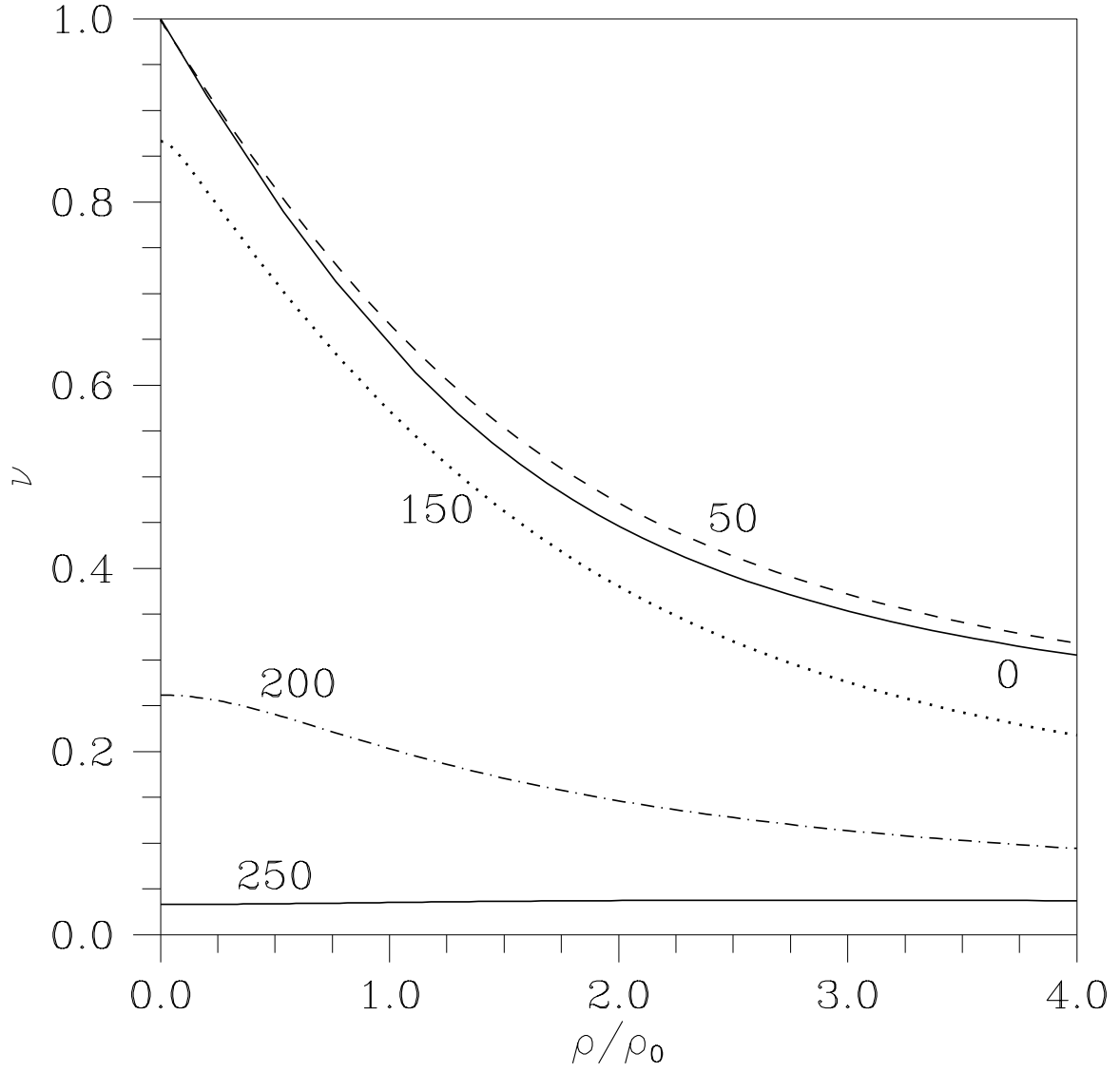


Figure 4. The mean sigma field, $\nu = \bar{\sigma}/\sigma_0$, as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

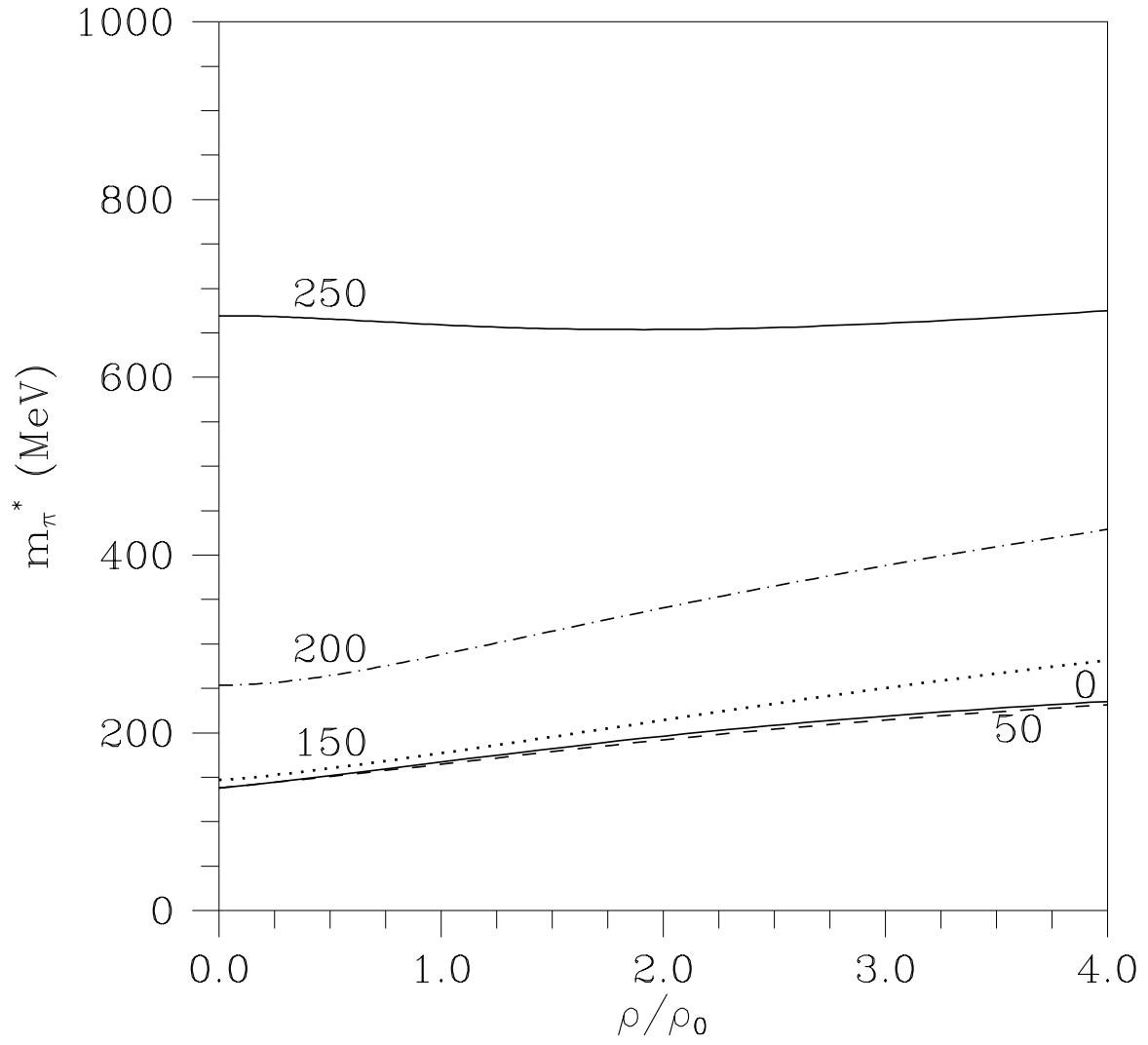


Figure 5. The pion effective mass, m_π^* , as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

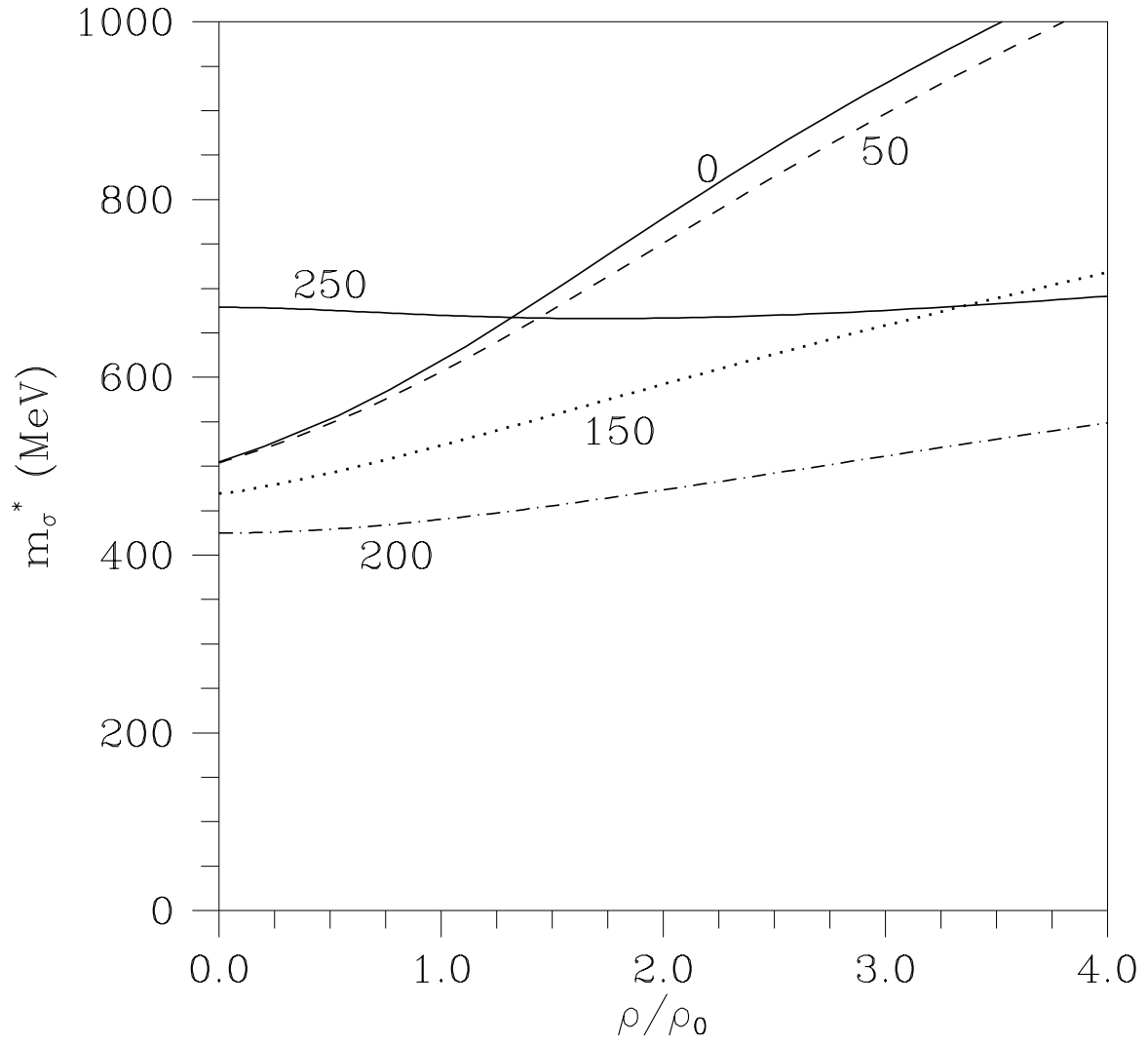


Figure 6. The sigma effective mass, m_σ^* , as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

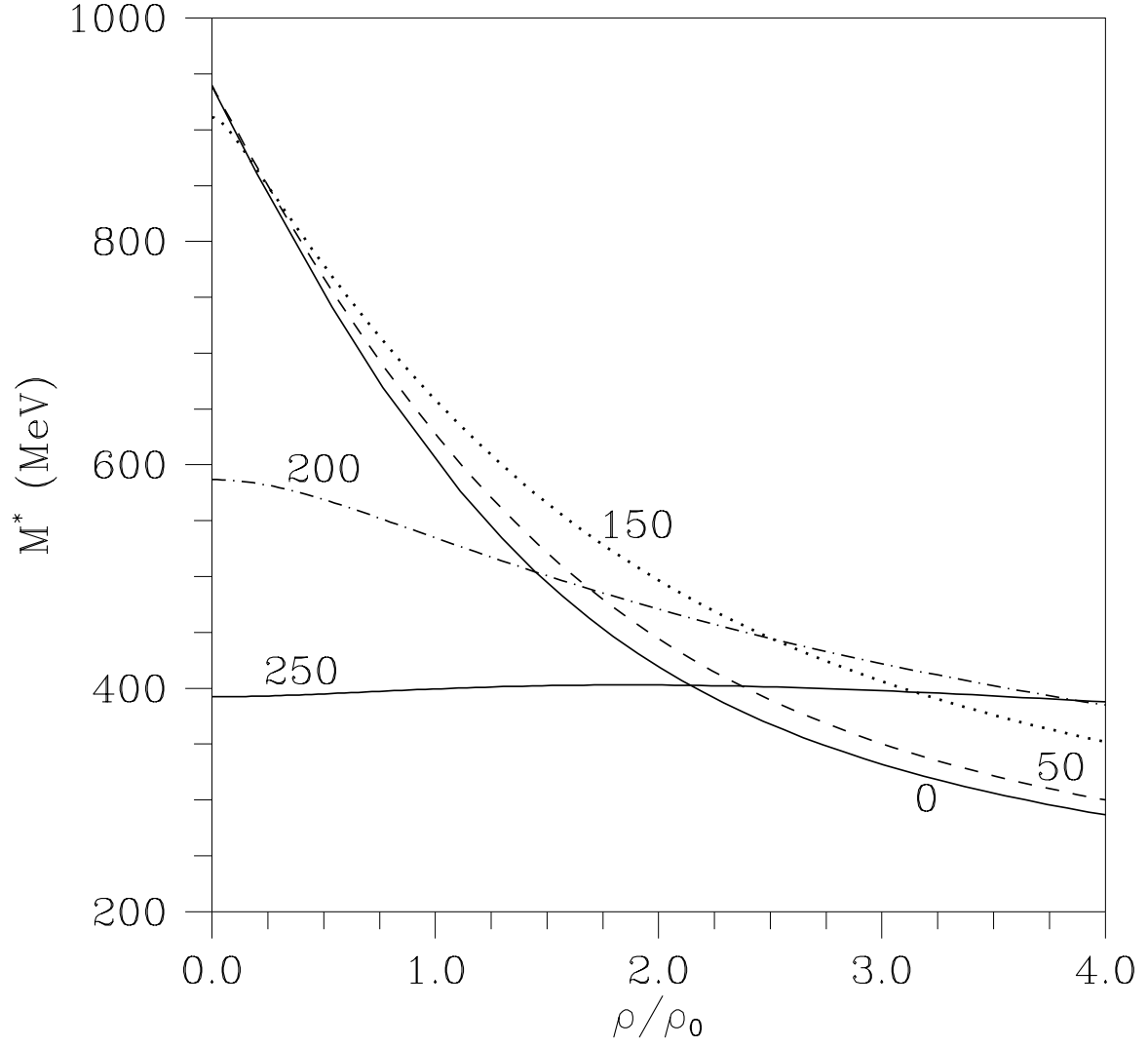


Figure 7. The nucleon effective mass, M^* , as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

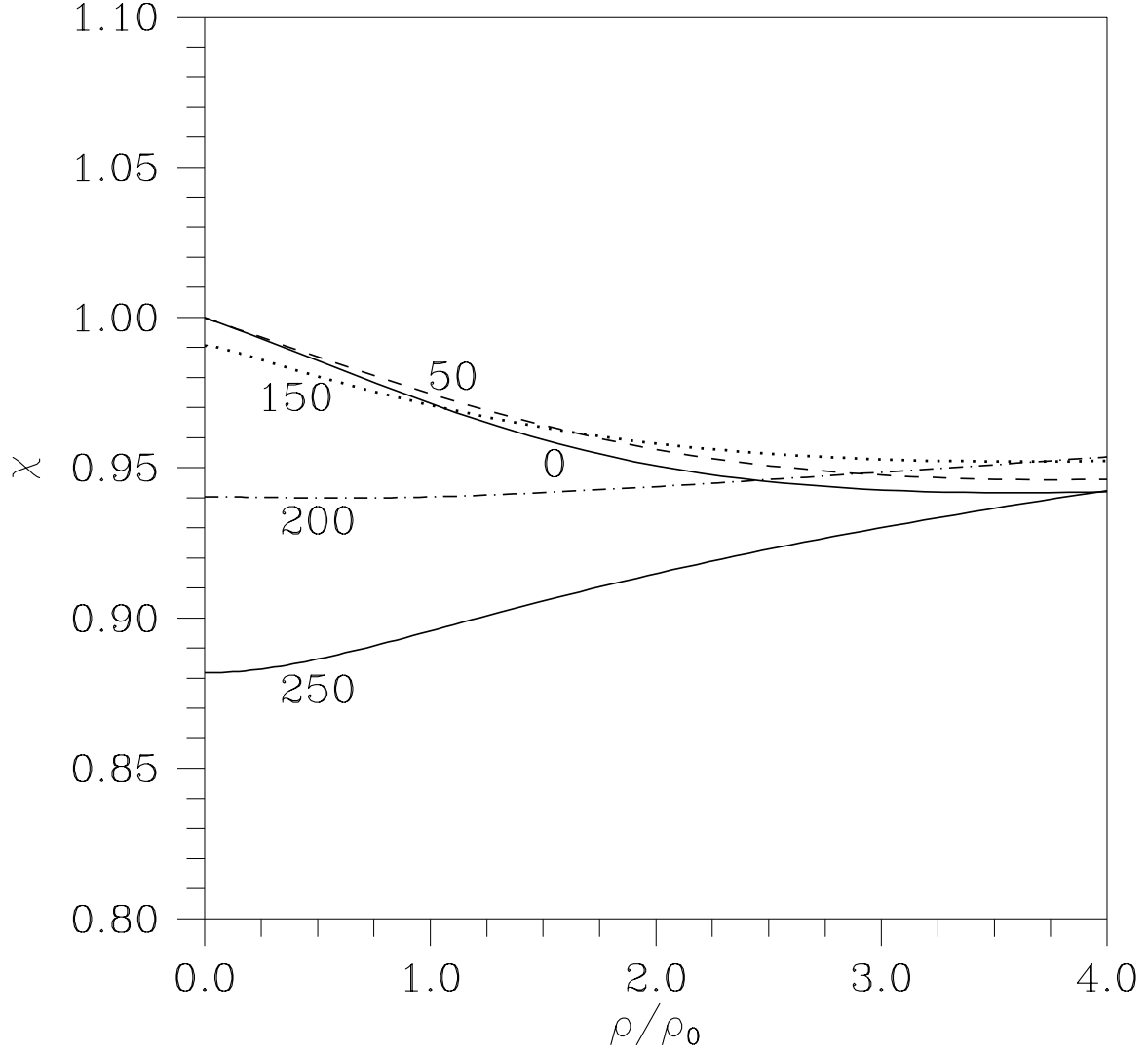


Figure 8. The glueball mean field, $\chi = \phi/\phi_0$, as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

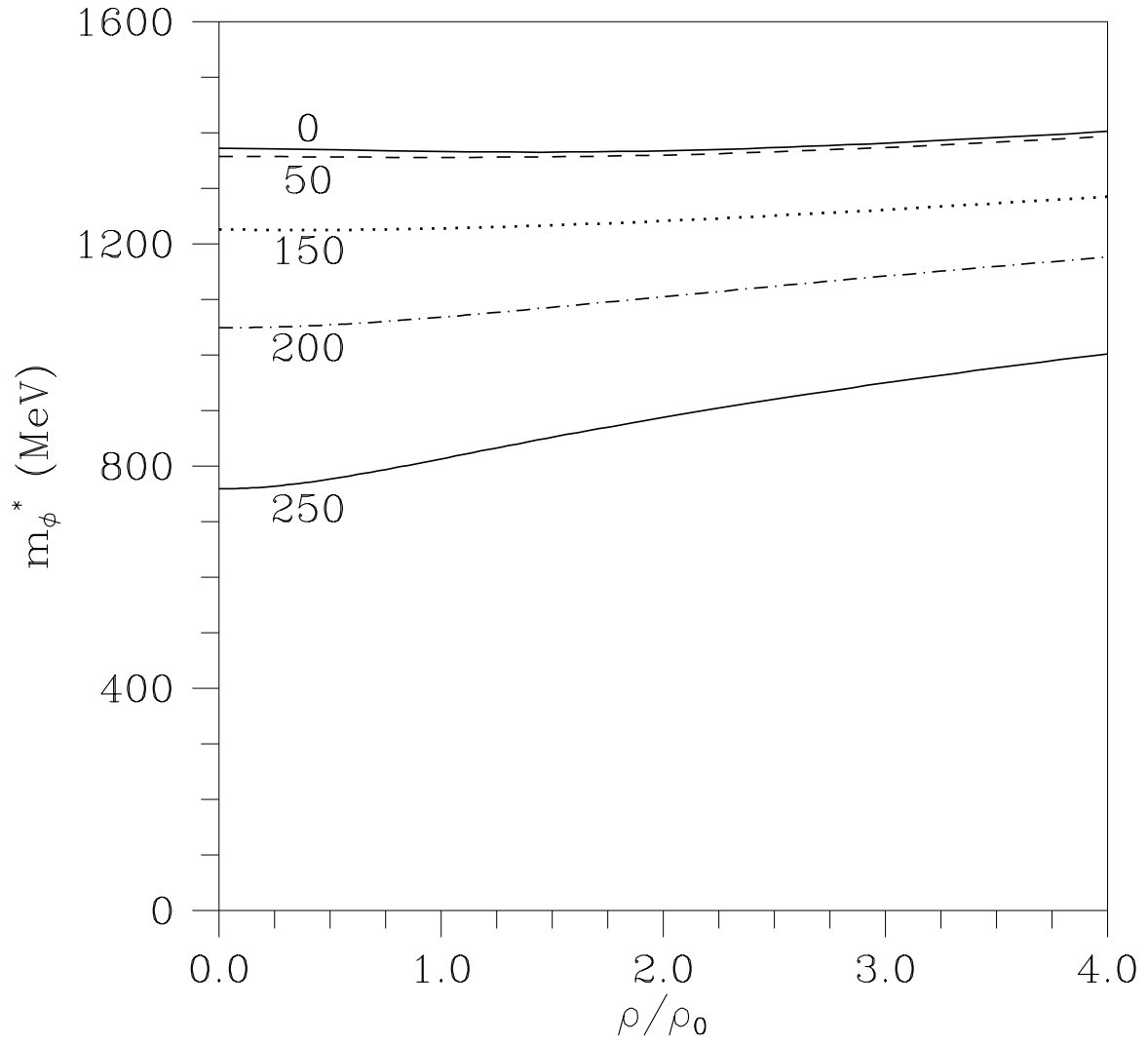


Figure 9. The glueball effective mass, m_ϕ^* , as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

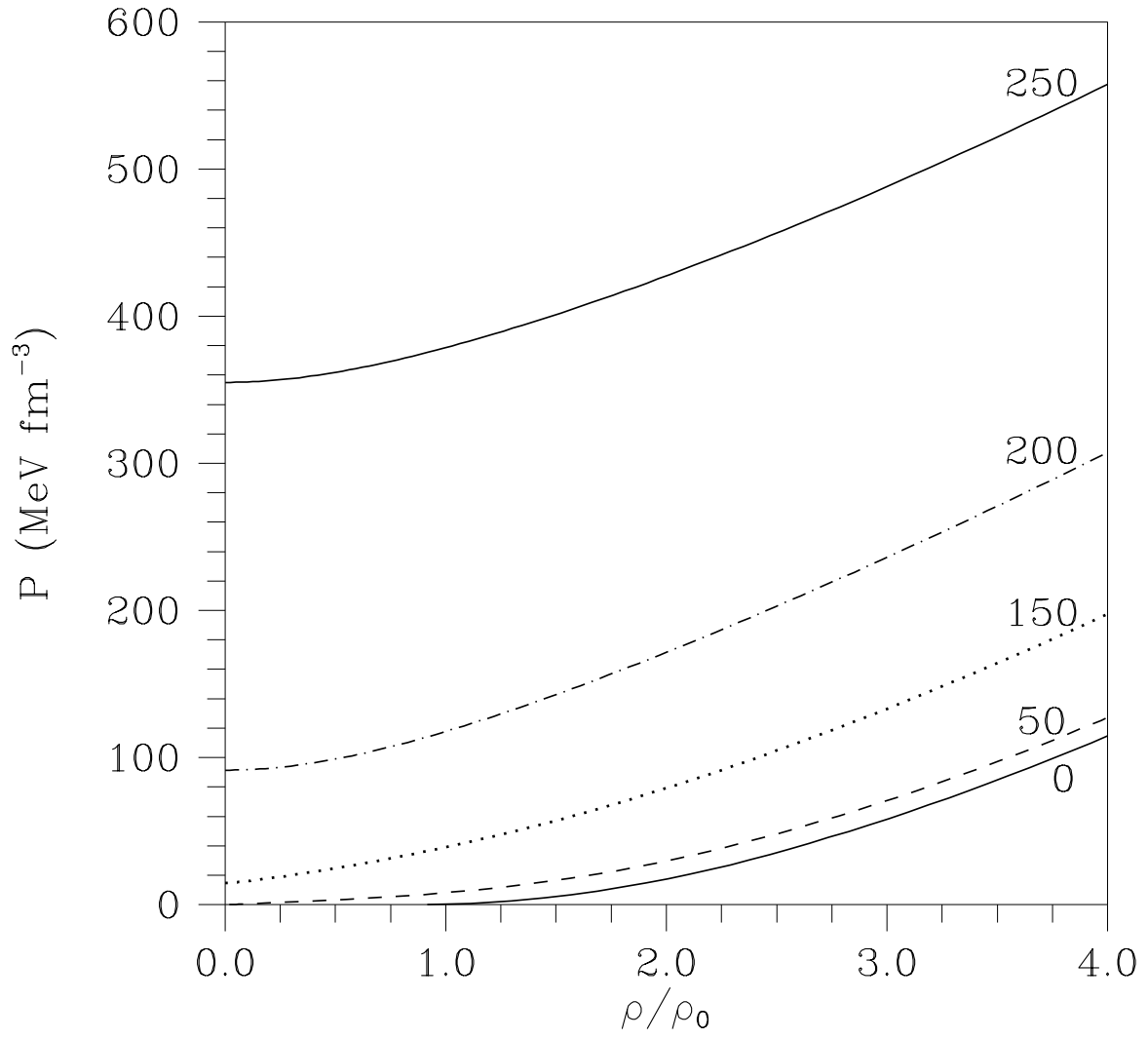


Figure 10. The pressure as a function of density for various temperatures (in MeV) with $\epsilon'_1 > 0$.

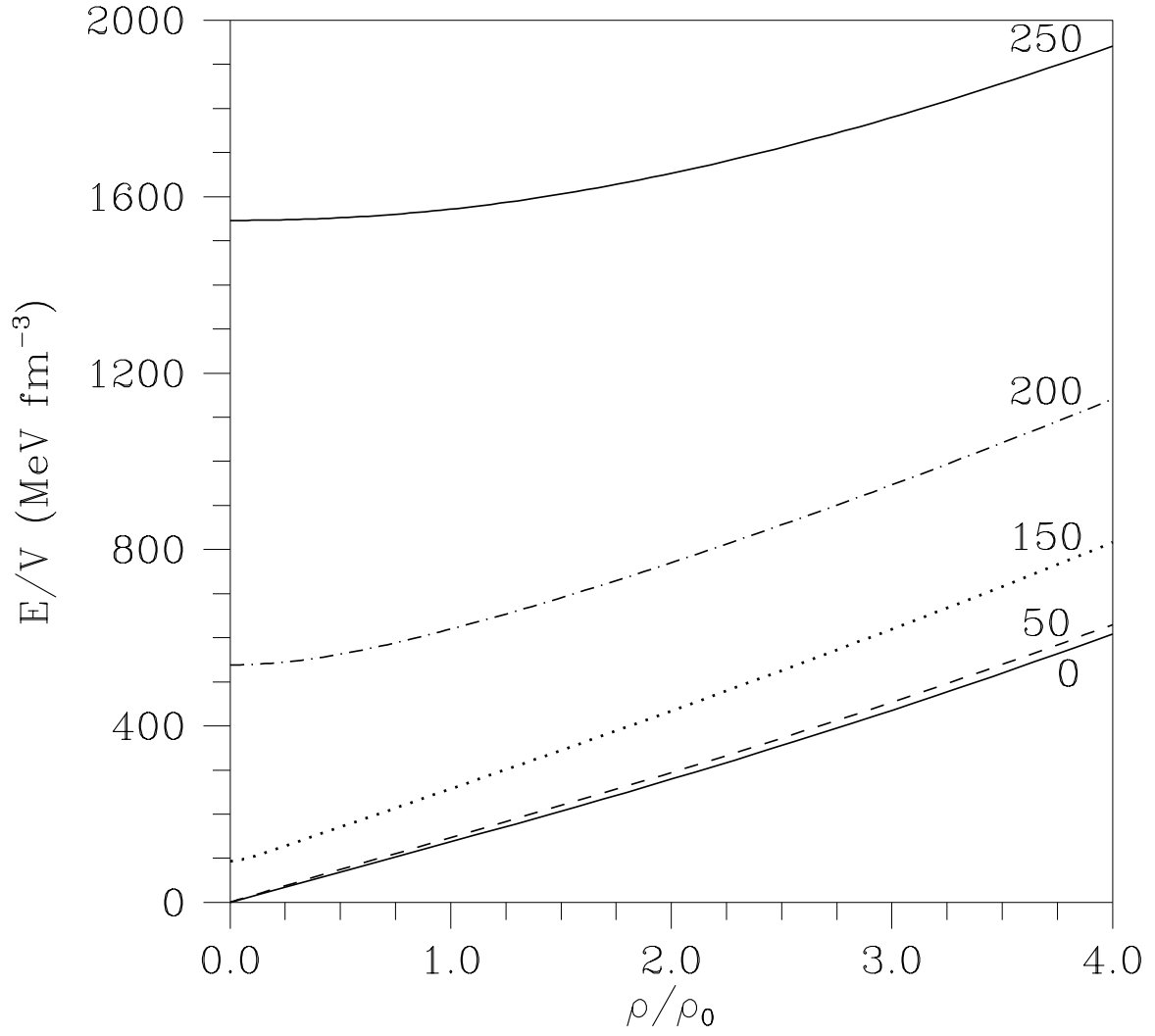


Figure 11. The energy density as a function of baryon density for various temperatures (in MeV) with $\epsilon'_1 > 0$.